POEMA Newsletter

October 2021

POEMA RESEARCHERS

Halfway through

The POEMA project is now in its third year. Initial results have been presented in our technical deliverables and publications on international journals.

POEMA ESRs are offered intense training programmes and excellent experts in the domain have been invited to deliver talks on a range of topics.

Luis Vargas, an early researcher (ESR7) at CWI was recently awarded the Best Poster Award at IPCO 2021, for his poster on his paper on Finite convergence of sum-of-squares hierarchies for the stability number of a graph. The detail of the poster is available at [1] and presentation video is available at [2].

Monique Laurent, researcher and management team member at CWI and a professor at Tilburg University, a member of POEMA network was recently elected as EUROPT Fellow 2021.

POEMA team was present in both SIAM OP 2021 [3] and SIAM AG 2021 [4]. We organized several mini-symposium on Positive Polynomials, Moments, and Applications including Bernard Mourrain (Inria), Victor Magron (LAAS-CNRS), Markus Schweighofer (University of Konstanz), Monique Laurent (CWI), Didier Henrion (LAAS-CNRS), Etienne de Klerk (University of Tilburg). Furthermore, POEMA ESRs gave talks at this virtual conference.

[1] https://cpb-us-w2.wpmucdn.com/sites.gatech.edu/dist/8/1446/files/2021/05/vargas-poster-4785.pdf

[2] https://mediaspace.gatech.edu/media/1_9xqzy7zg

[3] - https://www.siam.org/conferences/cm/conference/op21

[4] - https://www.siam.org/conferences/cm/conference/ag21

In this issue

POEMA Workshop 3

The event POEMA Workshop 3 was first planned at Inria, Sophia Antipolis, France. However, due to the situation of sanitary crises, this physical workshop has been changed to online event with a series of dates as following:

- Friday 29 January 2021
- Friday 05 February 2021
- Wednesday 17 February 2021

POEMA Workshop 3 continued to be designed to train the theoretical topics of POEMA programme. The workshop was organized around research presentations. High-profile speakers from Carnegie Mellon University, University of Birmingham, Georgia Tech, University of Massachusetts, Chinese Academy of Mathematics and System Sciences, CAS, LAAS-CNRS, University of Konstanz, presented the current cutting-edge research panorama in mathematical programming and global optimization. POEMA ESRs will also contribute talks in the workshop.

All tutorials including recorded video of the talks have been uploaded on POEMA website: https://bit.ly/3CBLCFM

Online Julia Training

The event of General Online Julia Training was organized on 16 April 2021 in preparation for Julia knowledge background of the Learning Week 2 in September 2021. This online training aimed to provide POEMA ESRs the following knowledge:

- The general soft skills including unix and git
- Lecture on general Julia and JuMP
- Exercise session on PMO DB and Math Opt. tools

Invited speakers who are experts in Julia were coming from Henallux and Université libre de Bruxelles, The Zuse Institute Berlin), LAAS – CNRS, University of Birmingham and Inria University Cote d'Azur. .

Tutorials of recorded videos of the talks are open to public and uploaded on POEMA website: https://bit.ly/2U7joTk

POEMA Learning Week 2

The global pandemic has been an unfortunate consequence of the COVID-19 outbreak, which has had significant impact on our project. However, thanks to virtual communication, such as online meetings, the progress of the project has not been seriously deprived and the productive work could still continue.

We celebrated POEMA Learning Week 2 in Toulouse, France from 13 – 17 September 2021 focusing on algorithms and modelling methods. This was the first physical event after a series of online events due to the pandemic.

The Learning Week 2 was organized around the following topics:

- 4 courses and tutorials to broaden the scientific scope of the ESRs with additional competencies, specific to the domain of polynomial optimization;
- scientific talks from experts in the domain of polynomial optimization and POEMA ESRs;
- Julia software training;
- and soft skills training on marketing the research experience, identifying personal skills and demonstrating them by storytelling

Besides POEMA members, invited speakers came from universities around Europe such as: UCLouvain, The Hong Kong Polytechnic University, University of Frankfurt, Technische Universität Braunschweig

It was such a nice occasion to have face to face meeting , work together and discuss on scientific topics on mathematical programming and global optimization.

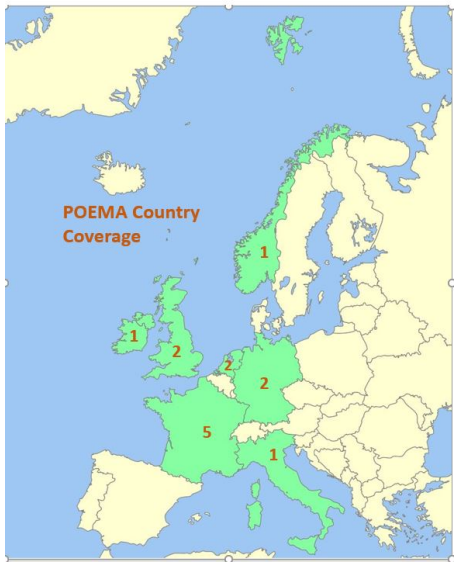
This was also the chance for ESRs to meet up again and enriched the networking not only internally but also externally with other ESRs and engineers outside POEMA who were present at the event and share similar interests in mathematics and optimization.

Tutorials of recorded videos of the talks are open to public and uploaded on POEMA website: https://bit.ly/3jAu7hl

Secondments

FOR MORE INFORMATION

Read more about ESRs' secondment in News and Events: https://bit.ly/38T86oS



There have been 4 secondments completion in POEMA since the beginning 2021 until October 2021. Due to the pandemic, some secondments were implemented online. The summary of these secondments is described as follows:

Trung Hieu Vu - University of Sorbonne

Hieu from University of Sorbonne had a remote secondment at University of Konstanz from February to April 2021. The purpose of this secondment was twofold. It was both an opportunity to make a connection between the secondee / experts who work at Konstanz University and also a chance to solve problems concerning the topic of computing sum of squares decompositions of polynomials with rational coefficients. During this remote secondment, the secondees had fruitful discussions on the problem of computing exact sum of squares decompositions of Hermitian complex trigonometric polynomials with Gaussian coefficients. Based on the discussions, they proposed a new algorithm to compute such sum of squares decompositions for polynomials that are non-negative on the unit circle. The algorithm has been implemented A complexity analysis of the algorithm has been started.

Felix Kirschner - University of Tilburg

Felix from University of Tilburg had a 3 month online secondment with LAAS-CNRS from April to June 2021. The purpose of the secondment was to further gather hands - on experience regarding the work in academia. To kick off a project of this kind, Felix and the team had to agree on a topic: the problem of finding optimal bounds on option prices, which can be cast as a generalized moment problem over the non - negative orthant with piecewise affine functions. Part of the learnings was to plan and coordinate meetings with a larger group of people and distribute tasks to be worked on until the next meeting. Ultimately, the purpose of

the secondment was to produce a piece of research worthy of publication.

Andrew Ferguson - University of Sorbonne

Andrew did 3 month of online secondment with University of Florence from May to August 2021. The purpose of this secondment is to explore new research directions relating to the problem of exact polynomial optimization from a different point of view. To create new interactions between nodes of the POEMA project that could lead to further collaboration. Andrew and Ettore (ESR5) at UNIFI consider the problem of deriving a certificate for non-negativity of a polynomial in the form of a sum of squares decomposition. They used the expertise of both teams, the team in Firenze and the team in Paris, to analyse the algebraic set defined by all possible sum of squares decompositions of a given non-negative polynomial

Ettore Turrati - University of Florence

In parallel with Andrew's secondment, Ettore had 3 month remote secondment with University of Sorbonne. The secondment allowed him to work on a intersection of the work carried out by ESR5 (Ettore) and ESR2 (Andrew), together with both supervisors Mohab Safey El Din and Giorgio Ottaviani. They worked on determining the degree of sum of squares. Namely, given a polynomial f that is a sum of k squares, we define SOS_k(f)={(l_1,...,l_k)|f=l_1^2+...+l_k^2} and we work on what is the degree of SOS_k(f). During the secondment, Ettore had online meetings every two weeks with the supervisors at SU and UNIFI, and a weekly meeting with Andrew Ferguson (ESR2). After the secondment, Ettore and the team are still working on the problem of the sum of squares and hope to have results to publish soon.

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Secondment at University of Florence by Andrew Ferguson (ESR2 from University of Sorbonne)

For the first half of my PhD project, as ESR 2, along with my advisors Prof. Mohab Safey El Din and Asst. Prof. Jérémy Berthomieu, we have studied practical algorithms for polynomial optimisation that return an exact representation of the infimum. Thus far, unlike the majority of member of the POEMA team, we have focused primarily on methods revolving around critical point methods and therefore, outside of the sum of squares (SOS) moment duality framework using the Lasserre hierarchy. However, my first secondment with the Florence node of POEMA, Prof. Giorgio Ottaviani and his student, ESR 5, Ettore Turatti, allowed me to delve into this world as well.

We study SOS decompositions from an algebraic point of view. For a given degree d and number of variables n, we consider the algebraic variety we call SoSk, the Zariski closure of the set of all polynomials that can be written as a sum of k squares, where k is at most n. With the Florence node's expertise on tensors and tensor decompositions, we were able to prove preliminary results on the degree of this object for the sum of one or two squares. This relates, by SOS/moment duality, to the algebraic degree of semi-definite programming defined and studied by Nie, Ranestad and Sturmfels. Currently, Ettore and I aim to extend our results to all k.

On the other hand, for a given polynomial f that can be expressed as a sum of k squares, we consider the variety of all possible SOS decompositions of f, in the ambient space of the coefficients of the SOS decomposition of f, SoSk(f). Again, we studied the degree of this object. Clearly, this variety has lots of groups acting on it. For example, when $k > 1$, one could simply permute the order of the decomposition elements. In fact, when f is quadratic - when the elements of its SOS decompositions are linear forms, we find that this variety is isomorphic to the orthogonal group of k elements. Thus, we can access its degree using the prior work of Brandt et al. who give this in a closed form. Furthermore, based on our experimental findings, we conjecture that there is a stabilisation meaning that this result holds also for all $d > 1$. We are currently exploring a couple of different methods to prove this result.

Up to now, the primary algebraic quantity studied for SOS decompositions was the degree of the so called central curve, the curve followed by numerical algorithms solving semi-definite programs. Alternatively, our work demonstrates another connection between algebra and SOS decompositions.

Throughout all our results, a key paradigm we used was a variation of one often seen in science, "guess and test". In this mathematical context, we adopt the advice, "guess then prove". By extensive computational experiments, we were able to deduce conjectures on the structures of the varieties described above. With these constructive results as a basis, rigorous proofs could then be developed.

Unfortunately, due to the global pandemic, this work had to take place entirely remotely. However, there are always silver linings. In hindsight, being English, Florence likely would not have been the ideal place to be during the weekend of the EURO 2020 final. But of course, once the situation allows, I look forward to returning to Florence, the site of the first POEMA workshop, as well as welcoming Ettore to Paris. Now, I am looking ahead to my last secondment at IBM research Dublin, my first experience in the industry and my first time visiting Ireland as this time it's due to be in person.

Secondment with University of Sorbonne by Ettore Turrati (ESR5 from University of Florence)

My first secondment was supervised by Professor Mohab Safey El Din, from Sorbonne University. Due to the sanitary crisis the secondment had to be done online. Although not being able to go to Paris and participate in the research activities of Sorbonne was a great loss, on the other hand this opened the possibility to have a project together with Andrew Ferguson, that would be in Florence, and my supervisor Giorgio Ottaviani.

We started the project by looking for a middle ground between the backgrounds of the two sides. We decided to study the decomposition of polynomials as sum of squares, more specifically, we were interested in understanding the degree of the variety of all sum of k squares of a general polynomial f that admits such decomposition. In a clearer way, the variety that we are interested in is

$$SOS_k(f) = \{(p_1, \dots, p_k) \in (Sym^d V)^{\times k} \mid f = \sum_{i=1}^{k} (p_i)^2\}.$$

The case d=1 corresponds to the Waring decomposition of f, one of the main problems studied in Florence.

Even if computing the degree of $SOS_k(f)$ is a simple problem to understand, tackling it showed to be much harder. We are still working on a proof that our conjuctered degree is indeed the degree of $SOS_k(f)$ and we plan to keep on working on it after the end of the secondment.

The secondment was a great opportunity to start projects different from my usual research and to create new research connections. I look forward to meeting Andrew and Mohab at the next event of POEMA in Toulouse so we can work on this project together in person.

Secondment with LAAS-CNRS by Felix Kirschner (ESR8 from University of Tilburg)

My secondment at CNRS - LAAS, Toulouse started in April 2021. Initially planned for October - December 2020, it had been postponed by half a year; a decision made with regard to the Covid situation and in hope of making a physical visit possible. As more and more cases were documented in Europe in spring it became clear that the secondment had to take place online.

To kick things off, I gave a presentation at the reading group at LAAS about my research, more specifically, a presentation based on the joint work with my supervisor on convergence rates for approximation hierarchies for the generalized moment problem (GMP) [3]. Afterwards, we had a Zoom meeting with Didier Henrion, Etienne de Klerk, Milan Korda, Jean-Bernard Lasserre and Victor Magron and agreed to work on the problem of data-driven options pricing in Finance (see, e.g. [1]). Interestingly, semidefinite programming techniques are usually not used Finance, but rather Monte-Carlo methods or for example the Black-Scholes model. Let $x \in \mathbb{R}_+$ be the price of an asset, e.g. a stock. An option on this stock is a derivative security, which gives the owner the right, but not the obligation to buy the stock at predetermined price k at a predetermined date T in the future. This means the pay-off function of this option is $\max(0, x_T - k)$. Finding bounds on the price of such an option given information about prices a_i for $i \in [n]$ of options on the same asset with different corresponding strikes k_i can be formulated as a GMP.

$$\begin{aligned} \sup / \inf_{\mu \in \mathcal{M}(\mathbb{R}_+)_+} & \int_{\mathbb{R}_+} \max(0, x-k) \mathrm{d}\mu(x) \\ \text{s.t.} & \int_{\mathbb{R}_+} \max(0, x-k_i) \mathrm{d}\mu(x) = a_i, \quad \text{for } i \in [n], \\ & \int_{\mathbb{R}_+} \mathrm{d}\mu(x) = 1. \end{aligned} \tag{1}$$

Loosely speaking, one is looking for a probability distribution of the stock price x that minimizes (resp. maximizes) the objective while meeting the known available data. A possible way to get bounds on the optimal value of (1) is to apply one of the Lasserre hierarchies (see [5], [6]). We wanted to prove the hierarchies converge to the optimal value of the problem we are considering. The main challenge in our framework was that the underlying set of the GMP was the range of prices the underlying assets the option is build upon can possibly attain. Mathematically speaking, is the nonnegative orthant. Taking $\mathbb{R}^n_+$ as the underlying set of the GMP introduces the difficulty of dealing with non-compactness. Most Positivstellensätze use compactness as a key ingredient, and Positivstellensätze are a common tool to prove convergence of the Lasserre hierarchy. To circumvent this problem we tried to prove that if the GMP is feasible then the infimum (resp. supremum) is attained and then applying the core variety procedure [2] to bound the support of the optimal measure in terms of the input data. We then identified cases in which our claim does not hold, i.e. instances of the problem that are feasible but in which the infimum is not attained. Equipped with this knowledge we were able to find a sufficient condition for the existence of an optimal measure with finite support in the univariate case. Using the bound we obtained this way we wrote software that takes as input empirical data, like known option prices with their respective strikes, and outputs optimal bounds for a option on a given strike price. The software is based on the Lasserre hierarchy of lower bounds, which we intend to complement by an implementation of the measure-based Lasserre hierarchy of upper bounds. Although the secondment is technically over, the team has agreed upon continuing to work on this project, study the multivariate case and further develop the software.

I learned a lot during the three months, gathered new insights in many interesting topics and am thankful for the opportunity to work with the great people at LAAS.

References

[1] Bertsimas, D., Popescu, I. (2002). On the Relation Between Option and Stock Prices: A Convex Optimization Approach. Operations Research. 50. 358-374. 10.1287/opre.50.2.358.424.
[2] Blekherman, G., Fialkow, L. (2018). The core variety and representing measures in the truncated moment problem. arXiv preprint arXiv:1804.04276.
[3] F. Kirschner, E. de Klerk, Convergence rates of RLT and Lasserre-type hierarchies for the generalized moment problem over the simplex and the sphere, arXiv preprint arXiv:2103.02924
[4] J. B. Lasserre, Moments, Positive Polynomials and Their Applications, Imperial College Press, 2009
[5] J.B. Lasserre, A new look at nonnegativity on closed sets and polynomial optimization. SIAM Journal on Optimization 21 (2011), no. 3, 864–885.
[6] J. B. Lasserre, The Moment-SOS hierarchy, arXiv:1808.03446