POEMA Newsletter

February 2021

In this newsletter:

POEMA RESEARCHERS

Progress of POEMA's ESRs in 2020

ESR1 has two papers for submission: "Sum of Squares Decompositions of Polyno-mials over their Gradient Ideals with Rational Coefficient and "On the solution existence and stability of polynomial optimization problems".

ESR2 has the paper, "On the computation of asymptotic critical values of polynomial maps and applications" describes progress towards realizing a polynomial optimization which yields an exact representation of the infimum of the input polynomial that can be used in practice.

ESR3 works on real varieties and semi-algebraic sets which are invariant under the action of some groups.

ESR4 has the paper with ESR5 on "Binary forms of suprageneric rank and the multiple root loci". The paper describes the strata of these varieties and explore the irsingular loc.

ESR5 is in a phase touching directly Polynomial Optimization. He considers the tensor space given by the tensor product of finitely many real vector spaces of finite dimension.

ESR6 works on an optimization problem, dealing with the minimization of certain classes of hypergraph-based polynomials over the standard simple.

ESR7 focuses on computing the stability number. He works on formulating then approximate the stability number with different techniques from the literature, such as via Lasserre hierarchy.

ESR8 studies the suitable approximation schemes. Of special interest is the convergence behavior of such approximation hierarchies, which often depends on the underlying feasibility set.

ESR9 studies the presentation theorems in real algebraic geometry, graded and border bases, duality of finitely generated algebras, conic duality, infinitely dimensional and truncated moment problem. The ESR focused on the Moment approximation, which has better properties than the Polynomial one.

ESR10 has the paper on "Computation of free non-commutative Gröbner Bases over Z with Singular: Letterplace".

ESR11 focuses his research during the first year on finite reflection groups, and in particular their invariant theory and their representation theory.

ESR12 explores the concept of generalized Augmented Lagrangians for the solution of semidefinite programs. This has been done with a particular focus on structures and difficulties which arise in the polynomial optimization context. The main two aspects which have been taken into account so far are the ill-conditioning and a low rank structure in the data.

ESR13 focuses on formulations of the truss topology optimization problem as a semi definite optimization problem with low-rank or approximate low-rank solutions and SDP problems with very low rank of the solution matrix. The ESR is writing the first joint paper with ESR12 under the title of "Penalty methods for low-rank semidefinite programming with application to truss topology optimization".

ESR14 works on problems using polynomial optimization and structure exploitation in the polynomial optimization problems arising from dynamical systems.

ESR15 investigates existing relaxations (including SDP relaxations) as well as their drawbacks. He has then focused on the possible exactness of second order moments relaxations and their computational tractability.

List of ESRs: https://bit.ly/2KtaAks
ESRs' papers: https://bit.ly/39G9ZnX
ESRs' blog: https://bit.ly/3qAj74z
ESRs' work in POEMA: https://bit.ly/2XRtFQw

POEMA Learning Weeks 1

POEMA's 1st Learning Weeks were organized online due to the pandemic. The learning weeks lasted from 27 May to 16 September 2020.

The program included short introductory courses (1 - 1,5 hours) to broaden the scientific scope of the ESRs, and to equip the Phd students with additional competencies, specific to the domain of polynomial optimization. It allowed the ESRs to acquire a common knowledge, to be shared by the network community. These courses addressed different aspects of effective algebra, convex geometry, and polynomial optimization and were provided by POEMA members or other high-profile speakers in Europe and the United States. All the seminars were recorded and published on POEMA website. This online event received 200 registrations and 60 participants on average attended each course during the Learning Weeks.

The invited speakers of the learning weeks were:

Description of POEMA learning weeks by participants:

- Didier Henrion (LAAS-CNRS) – courses on Polynomial Optimization
- Edouard Pauwels (Institut de Recherche en Informatique de Toulouse) – courses on Christoffel-Darboux Kernels
- Cordian Riener (UiT The Arctic University of Norway) – courses Symmetries in algorithmic questions in real algebraic geometry
- Mihai Putinar (University of California at Santa Barbara)) – courses on Moments

Link of POEMA Learning Week 1 event with available tutorials: https://bit.ly/2OlPBSl

POEMA Consortium during the workshop

POEMA Worskhop 2

Similarly to the Learning Weeks, there has been in changes in plan of the Workshop 2. Due to sanitary crises, this physical workshop has been also changed to online event with a series of dates:

- Tuesday 20 October 2020
- Thursday 26 November 2020
- Friday 27 November 2020 (POEMA ESR Internal Day)
- Friday 11 December 2020

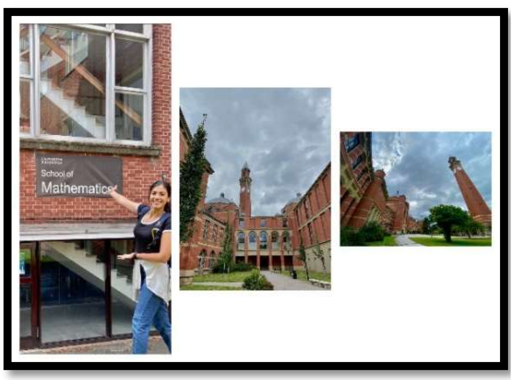
The 2nd Workshop was dedicated to the foundations of polynomial optimization. There were talks in a broad range of topics by high-profile speakers from reputed universities and companies in polynomial optimization domain, presenting the current cutting-edge research panorama in mathematical programming and global optimization.

We had 60 participants on each day of the workshop. The discussion was open and fruitfully achieved. POEMA ESRs presented the progress of their research with active discussion within the consortium and valuable advices from the professors/ advisors of the network.

Link of POEMA Workshop 2 event with available tutorials: https://bit.ly/3tEJPeE

POEMA's secondments in 2020

Shared working space of Arefeh and Soodeh during the secondment at FAU.



Arefeh at University of Birmingham during her secondment.

FOR MORE INFORMATION

Read more about ESRs' secondment in News and Events: https://bit.ly/38T86oS

There have been 5 secondments in POEMA in 2020. Due to the pandemic, some secondments were implemented online. The summary of these secondments is described as follows:

Alejandro González Nevado - University of Konstanz

Alejandro from University of Konstanz had a secondment at University of Florence (UNIFI). The purpose of his secondment was to acquire new perspectives about tensor decomposition and algebraic geometry. After the secondment, he started to prepare a paper with Ettore (ESR5 at UNIFI). The three month secondment has brought him new knowledge, experiences and helped him better in his current research.

Sebastian Debus - UiT The Arctic University of Norway

Sebastian from UiT The Arctic University of Norway had 2 month online secondment with University of Konstanz (Germany). The purpose of the secondment was to obtain an understanding of the moment problems and non-negativity questions which is a central research topic in Constance and to explore these in equivariant situations.

Arefeh Kavand - Friedrich-Alexander-Universitaet Erlangen (FAU)

Arefeh from Friedrich-Alexander-Universitaet Erlangen (FAU) completed 3 month secondment at University of Birmingham (UK). She worked closely in collaboration with ESR13, Soodeh Habibi on solving SDP problems using "Interior point method" and to design proper preconditioners for solving Hessian system. In this secondment she aims to prove some features of the new preconditioners theoretically which has been shown numerically. These features are tested on "truss problems" using Penalty/Barrier Multiplier method by using so-called preconditioners.

Soodeh Habibi - University of Birmingham (UoB)

Soodeh from University of Birmingham spent 3 months at FAU (Germany) during her secondment. Each team are using different approaches for solving large-and-sparse low-rank linear Semidefinite Programs (SDPs). These approaches are by using Interior-Point (IP) method in UoB and Primal-Dual Penalty/Barrier Multiplier (PD-PBM) method in FAU. In UoB we have developed a preconditioner for solving SDPs. The main purposes of this secondment were trying to enhance the effectiveness of these preconditioners on both IP and PD-PBM methods, finding the similarities of these two methods on SDPs more specifically on Truss topology problems, and start writing their joint paper.

Tobias Metzlaff - Inria

During this 3 month secondment to University of Tromso (Norway), Tobias was suggested to use the so far obtained research results to improve a bound for the chromatic number of lattices originating from root systems, which are the infinite families An, Bn, Cn, Dn and the finite families En, Fn and Gn.

The bound for the chromatic number can be obtained from an optimization problem on functions, that were studied by EH and TM before the secondment. And The work on the proof of the bound for Dn and also the proof of these theorems is in progress and will continue after the secondment.

Tobias also participated in an online conference organised by the hosting institution for the community of Norwegian mathematicians.

The workgroup established during the secondment, collected the obtained results in an article and plans to publish them in the near future.

On the Penalty/Barrier Multiplier method for solving linear Convex SDPs

Arefeh Kavand - Friedrich-Alexander-Universitaet Erlangen (FAU)

In the first months of the project we (ESR12, Prof Michael Stingl) have been exploring the concept of generalized Augmented Lagrangians for the solution of semidefinite programs. This has been done with a particular focus on structures and difficulties which arise in the polynomial optimization context. The main two aspects which have been taken into account so far are the ill-conditioning and a low rank structure in the data.

In order to face the problem of ill-conditioning was approached, a general class of barrier functions (so called penalty-barrier-functions) was suggested, which provides an upper bound for the eigenvalues of the Hessian of the Augmented Lagrangian. While such an approach has been reported in literature before, in our project, we combine it with a regularization technique, which provides - in addition - a uniform lower bound for the Eigenvalues.

Another important aspect is that - when applying penalty-barrier-functions in the context of semidefinite programming - the computational complexity for assembling the Hessian matrix is one order of magnitude higher than in comparable approaches using modified barrier functions or even the well-known log det- function as used in interior point methods. In our project, we were able to show that, when combining the Penalty-Barrier-Concept with iterative solutions methods for the solution of arising Newton-type systems, this disadvantage can be fully eliminated. This is of particular interest for polynomial optimization problems, as for tight approximations, often very large SDPs are obtained. In this cases assembling the full Hessian of the Augmented Lagrangian is expensive and sometimes the Hessian cannot even be stored.

Despite Covid 19 and all the restrictions, fortunately, I could spend my first research stay at the University of Birmingham (UoB). I did the half of it remote and the rest of it in person on August-September 2020.

The research teams at FAU and UoB have close scientific interaction on the research work. We have started to organize meetings between the two universities on a regular basis (approximately biweekly). In these meetings ideas between me and ESR13 (and our advisors) on the development of optimization solvers taking structures specific to polynomial optimization problems into account are exchanged. This includes a common data and software basis, maintained by the two groups. A particular focus of these meetings so far was to identify common structures in interior point methods and primal-dual methods based on Augmented Lagrangians. It turned out that the linear systems which have to be solved at the core of both methods have a mathematically equivalent structure. Thus it seems possible to develop iterative methods and, in particular, preconditioners, which can be applied in the framework of both solution concepts. These methods have been tested by means of problems from truss topology design.

Our scope will be extended to more general polynomial optimization problems and to keep working on preconditioners helping to cope with the high computational complexity expected for large polynomial optimization problems provided by our partners in future.

Besides, it was such a great opportunity to gain the life experience and to know more about the preconditioners by reading related papers and discussing ESR13 about the details and proofs of the lemmas and theorems, which was totally productive and useful to me.

Secondment report: Tobias Metzlaff (ESR 10) at UiT Tromsø.

Tobias Metzlaff – Inria

Tromsø, or as we learned to call it, the "Cote d'Azur du Nord" is the most northern POEMA location and I had the great pleasure to spent three months from November 2020 to January 2021 there to absolve my first secondment.

The name of the host institution is UiT Tromsø, but it is commonly known as the arctic university. Upon my arrival, I was welcomed by a cold breeze, snow and a sparsely populated city. Things, that are not to be found in southern France, where I was coming from. Following the restrictions made by the Norwegian government, I spent the first 10 days in self isolation. During this time I presented my so far carried out research work to hosting professor Cordian Riener and the members of his work group from the department of mathematics and we discussed possible applications.

Philippe Moustrou, a postdoc at the department of mathematics, suggested to use our know-how to compute chromatic numbers. The goal was, to find a lower bound for the number of colors needed to "paint" the Euclidean space, such that two points with distance one don't share the same color. Clearly, this number depends on the metric of the space and particularly nice results are known and can be obtained when it originates from root lattices. Together with my supervisor Evelyne Hubert, we studied several examples, investigated the necessary theory and found new, exciting approaches to this problem.

Apart from a great community of international scientists, Tromsø also offers amazing natural sights. The main island is surrounded by mountains and the northern sea. From there, one can start whale-watching tours, hiking trips and follow all kinds of winter sports activities. A truly unique event in these latitudes is the polar night or "Mørketidens" from mid December to mid January. During this time of the year, the sun remains below the horizon and apart from some hours around noon, the days are completely dark. Therefore, the chance of observing the aurora borealis is much higher and I was lucky to be a witness of this amazing spectacle a couple of times.

Fitting to the season, the department of mathematics organized a small semi-virtual conference, the "Mørketidens Mattemøte". Many members of the community of Norwegian mathematicians participated, including Hans Z. Munthe-Kaas, who gave a talk about affine Weyl groups and interpolation with multivariate Chebyshev polynomials. A perfect opportunity to ask questions about his work, which is related to ours.

The work group, that established during the secondment, will continue the common work and stay in close contact. For me personally, this stay abroad was very helpful, as it allowed me to look at my own research from a new point of view and to establish new connections. I am looking forward to return to Tromsø, may it be to participate in the MEGA conference or just to see what more it has to offer during other times of the year.

An Interior-Point Method for Low-Rank Semidefinite Programming

Soodeh Habibi, University of Birmingham, England

Solving semidefinite programs (SDP) present many challenges which require various techniques to overcome. Among these challenges, we are focusing on solving large-and-sparse low-rank semidefinite programs by using a variant of an interior point method. Interior-point methods are techniques used to convert the conic problems into a sequence of unconstrained problems. The bottleneck in an interior-point method consists in assembling and solving the so-called Schur complement equation, a large system of linear equations with a positive definite matrix. As the matrix is large and sparse, the Schur complement equation can be solved by using an Krylov type iterative method such as the conjugate gradient (CG) method. The convergence rate of such a method is usually slow due to ill-conditioning of the matrix. In fact, the Schur complement matrix becomes increasingly ill-conditioned as the interior-point method makes progress towards the solution. Hence, to solve the linear system, instead of CG we are using a modified version which is called Preconditioned Conjugate Gradient (PCG). Efficient preconditioners allow PCG to converge to a solution of the linear equation in a few iterations, independent of the ill-conditioning of the Hessian matrix.

Development of efficient preconditioners is very much problem dependent. In our research, we are focusing on SDP problems with (expected) very low rank of the solution matrix. Using this and further assumptions, we have proposed two new preconditioners, inspired by the article by Richard Y. Zhang and Javad Lavaei entitled \Modified Interior-Point Method for Large-and-Sparse Low-Rank Semidefinite Programs". Numerical experience shows that our modified preconditioners are more efficient than those by Zhang and Lavaei. Moreover, our interior-point method with the new preconditioners is substantially faster than standard available software for SDP, such as MOSEK. Our main focus is on solving large-scale problems resulting from truss topology optimization.

The figure below shows a comparison of MOSEK with our Matlab implementation which is called myIP. This figure demonstrates much smaller computational complexity of myIP as compared to MOSEK and allows us to predict that we can solve problems of much bigger dimension than standard software. The two teams UoB and FAU have been collaborating to investigate commonalities present in both interior point methods and primal-dual methods based on Augmented Lagrangians. As a consequence of this collaboration, part of which took place during my secondment at FAU, we have observed that the linear systems central to both methods have a mathematically equivalent structure. Therefore, it seems possible to develop iterative methods and, in particular, preconditioners, which can be applied in the framework of both solution concepts.

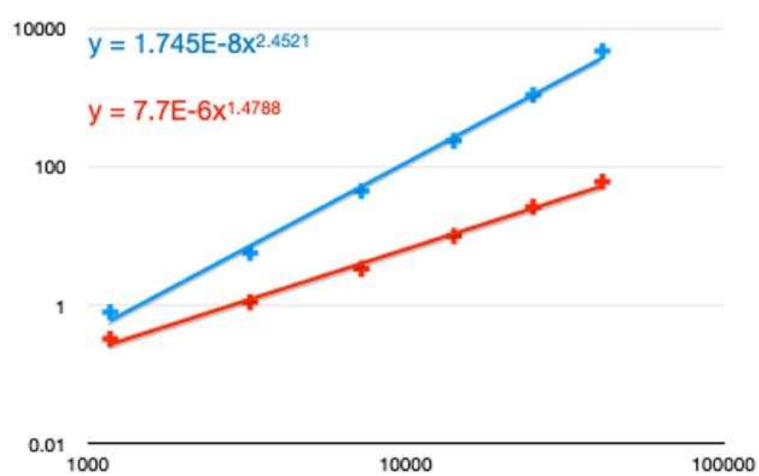


Figure 1 - Mosek vs myIP CPU time for truss topology problems. The x-axis and y-axis represent the number of variables and CPU time in seconds, respectively.