Polynomial matrix inequalities in structural optimization

Abstract

The purpose of this talk is to introduce our recent contributions to the application of polynomial matrix inequalities in designing high-performance elastic structures. The presentation is organized into two parts covering an industrial application of large-scale linear semidefinite programming [1] and the subsequent generalization to polynomial matrix inequalities [2] that is currently mainly of academic interest.

In particular, the first part addresses thin-walled composite tubes made of high-performance carbon fiber composites. Such structures display excellent stiffness-to-weight ratios but, at the same time, suffer from wall instabilities that restrict their use in dynamic environments. To mitigate this limitation, we develop a linear semidefinite programming formulation to design an efficient modular truss internal structure. The formulation seeks for a minimum-weight reinforcement with lower bounds on the fundamental free-vibration eigenfrequency of the reinforced product and its stiffness during the manufacturing process. We discuss the whole production pipeline of a composite machine tool component covering (i) the optimal design and its acceleration by Schur decomposition techniques, (ii) prototype manufacturing by conventional 3D printing, and (iii) verification and (iv) validation of the optimal design.

The developments in the first part of the talk apply solely to truss structures capable of transmitting only axial forces, which manifests in the linear dependence of stiffness and mass matrices on the optimized variables. Incorporating bending effects leads to a polynomial dependence, which renders the problem non-convex even for the basic minimum-compliance scenario. In the second part of the talk, we develop a strategy that facilitates computing all guaranteed globally optimal solutions to such problems. Our approach builds on an equivalent non-linear semidefinite programming reformulation with a semi-algebraic feasible set, which is solved using the Lasserre moment-sum-of-squares hierarchy. Moreover, we introduce a simple procedure for projecting the relaxed solutions onto the feasible set of the original problem to establish a sequence of feasible upper bounds. When combined with lower bounds obtained from the hierarchy, these provide a simple sufficient condition of global ε-optimality. We prove that this condition is tight in the limit for problems with the global solutions forming a convex set. Using a set of small-scale examples, we show that the hierarchy usually converges in a finite (and fairly small) number of steps.

Acknowledgement. This work was supported by the Czech Science Foundation project No. 19-26143X.

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[2] Tyburec, M., Zeman, J., Kružík, M., & Henrion, D. (2021). Global optimality in minimum compliance topology optimization of frames and shells by moment-sum-of-squares

hierarchy. Structural and Multidisciplinary Optimization, 64(4), 1963–1981.
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