

Talk: Projective limit techniques for the infinite dimensional moment problem

Speaker: Maria Infusino (University of Cagliari)

Abstract: In this talk we focus on the following general version of the moment problem: when can a linear functional L on a unital commutative real algebra A be represented as an integral w.r.t. a positive Radon measure on the space $X(A)$ of all homomorphisms on A equipped with the Borel sigma-algebra generated by the weak topology? In a joint work with Salma Kuhlmann, Tobias Kuna and Patrick Michalski we construct $X(A)$ as a projective limit of all $X(S)$ with S finitely generated subalgebra of A , so to be able to exploit the results for the classical finite dimensional moment problem in the infinite dimensional case. In fact, we show that the existence of a Radon representing measure μ_S for L restricted to each S s.t. $\{\mu_S\}_S$ fulfils the Yamasaki condition is equivalent to the existence of a representing measure for L on A but defined on a smaller σ -algebra than the Borel one, namely the cylinder σ -algebra on $X(A)$ carried by the projective limit construction. Such a measure can be extended to a Radon representing measure whenever $\{\mu_S\}_S$ fulfills the well-known Prokhorov condition, which also implies the Yamasaki condition. These results allow us to establish infinite dimensional analogues of the classical Riesz-Haviland and Nussbaum theorems as well as a representation theorem for linear functionals non-negative on a “partially Archimedean” quadratic module of A . Applying our results to the algebra of polynomials in infinitely many variables, we can retrieve the solution to the moment problem for constructibly Radon measures given by Mehdi Ghasemi, Salma Kuhlmann and Murray Marshall in 2016.