

Moment problem in infinitely many variables

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Abstract

In this talk, we present the moment problem for the polynomial algebra $A := \mathbb{R}[x_i \mid i \in \Omega]$ in an arbitrary number of variables x_i , $i \in \Omega$. We introduce constructibly Radon measures on its character space \mathbb{R}^Ω , and proceed to investigate their relationship to positive linear functionals on A . The main tool is to exploit the localisation $B = B_\Omega := \mathbb{R}[x_i, \frac{1}{1+x_i^2} \mid i \in \Omega]$ of A . We show that positive linear functionals on B_Ω correspond bijectively to constructibly Radon measures on \mathbb{R}^Ω , and that the moment problem for A_Ω reduces to understanding the extensions of a positive linear functional on A_Ω to a positive linear functional on B_Ω .

We observe that A (resp. B) is the inductive limit of the \mathbb{R} -algebras A_I (resp., B_I), I running through all finite subsets of Ω . So many questions about A and B reduce to the case where Ω is finite. This last observation is exploited and formalised as the *projective limit approach* to the moment problem (see M. Infusino 's talk), connecting in particular constructibly Radon measures to cylindrical measures. This is joint work with Mehdi Ghasemi and Murray Marshall.

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